Multicell Converters Hybrid Sliding Mode Control

O. Benzineb[1,2], F. Taibi[3], M.E.H. Benbouzid[1], M.S. Boucherit[3] and M. Tadjine[3]

Abstract–*This paper deals with hybrid sliding mode control of multicell power converter. It takes into account the hybrid aspect of the conversion structure which includes the converter continuous and discrete states. The basic idea used in this paper is to consider the interconnected systems that represent the hybrid model and to generate commutation surfaces based on a Lyapunov function that satisfies asymptotic stability. Simulations are carried-out on a two-cells converter to assess the performances and the robustness of the synthesized controller.*

***Keywords**: Multicell converter, hybrid system, sliding mode control, interconnected systems, robustness.*

Nomenclature

E	=	DC source;
V_s	=	Output voltage;
v_{Ck}	=	Floating voltage;
I (i)	=	Load Current;
u_k	=	Binary switch;
x	=	Continuous states variables vector;
u	=	Discrete control sequences vector;
Tij	=	transition conditions;
X	=	Average states variables vector;
α	=	Switching duty cycles vector;
C	=	Capacitor;
R	=	Load resistance;
L	=	Load inductance;
p	=	Converter number of cells.

I. Introduction

Hybrid systems constitute a multi-disciplinary area which arises during the last decade and extends between the limits of computer science, applied control engineering and mathematics. A hybrid system is a mathematical model able to represent some complex physical systems with hierarchic structure and made up of discrete and continuous subsystems which communicate and interact with each other. Switching circuits in power electronics are a particularly good candidate for hybrid analysis because they are inherently hybrid in structure. Under this hybrid model the system has discrete inputs, continuous outputs, and disturbances that are either continuous, as in a changing load or source, or discrete, as in a fault condition for a particular switch.

Among these switching circuits, multicell converters are based on a series-association of elementary commutation cells. This structure, which appeared at the beginning of the 90's, allows voltage constraints sharing by the commutation cells series-connected. By the way, the waveform harmonic content is greatly improved [1-6].

Besides, modeling is a very important step for control laws and observers synthesis. As modeling accuracy depends on the required goals, one can find several model kinds for the same process and the choice among those will depend on its use and on the control objective. For the control or the observer synthesis, the selected model must be sufficiently simple to allow real time control (or observation) but enough precise to achieve the desired behavior. Multicell converter modeling is generally difficult. Indeed, Indeed, it carries continuous variables (currents and voltages) and discrete variables (switches, or discrete location) [7-8]. In the available literature, three types of models could be found: *1)* The *average model* based on calculating average value of all variables over one sampling period. However, this model cannot represent the capacitors terminal voltage natural balancing; *2)* The *harmonic model*. It is based on the calculation of the voltage harmonic phases and amplitudes by considering the charging current in steady-state operation; *3)* The *exact or instantaneous model* which takes into account time-evolution of all variables including the switch states (discrete location). This model is difficult to use as controllers and observers design is impossible since the converter is not a continuous system but the combination of continuous and discrete systems [2-3].

Hybrid modeling will allow multicell converters using analysis and synthesis powerful tools for a better exploitation of controller possibilities [9]. This paper proposes the hybrid modeling of a two-cell converter which will be afterwards controlled using sliding modes. In this case, a stability study of the closed-loop system is carried-out.

II. Multicell Converters Briefly

Multicell converter consists of cells, where each one contains two complementary power electronics components and it can be controlled by a binary switch u_k (Fig. 1).

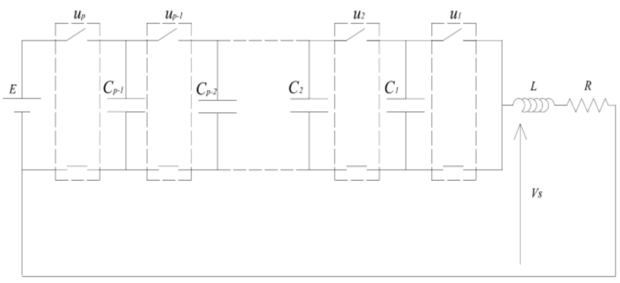


Fig.1. *p*-cells converter.

This signal is equal to 1 when the cell upper switch is conducting and equal to 0 when the lower complementary switch is conducting. These cells are associated in series with an *RL* load and separated by capacitors that can be considered as continuous sources [1], [10].

The converter has $p-1$ floating voltage sources. In order to ensure normal operations, it is necessary to guaranty a balanced distribution of the floating voltages ($v_{Ck} = kE/p$). The output voltage V_s possesses p voltage levels ($0, E/p, \dots, (p-1)E/p, E$) [10].

The system model can be obtained using electrical laws represented by p differential equations giving its state space representation with the floating voltages v_{Ck} and the load current i as state variables.

$$\begin{cases} \dfrac{dv_{C_1}}{dt} = \dfrac{1}{C_1}(u_2 - u_1)\, i \\ \vdots \\ \dfrac{dv_{C_{p-1}}}{dt} = \dfrac{1}{C_{p-1}}\left(u_p - u_{p-1}\right) i \\ \dfrac{di}{dt} = -(u_2 - u_1)\dfrac{v_{C_1}}{L} - (u_3 - u_2)\dfrac{v_{C_2}}{L} - \\ \qquad \cdots - \left(u_p - u_{p-1}\right)\dfrac{v_{C_{p-1}}}{L} - \dfrac{R}{L} i + u_p \dfrac{E}{L} \end{cases} \tag{1}$$

The above system can re-written as

$$\Sigma_{Converter} : \dot{x} = A\, x + B(x) u \tag{2}$$

where

$$- \begin{cases} x = [v_{C_1}, v_{C_2}, \dots, v_{C_p}, i]^T \\ u = \left[u_1, \dots, u_p\right]^T \end{cases}$$

$$- \ A = \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & 0 & \vdots \\ 0 & \cdots & -\dfrac{R}{L} \end{bmatrix}$$

$$- \ B(x) = \begin{bmatrix} -\dfrac{i}{C_1} & \dfrac{i}{C_1} & 0 & \cdots & 0 \\ 0 & -\dfrac{i}{C_2} & \dfrac{i}{C_2} & \cdots & 0 \\ \vdots & 0 & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & -\dfrac{i}{C_{p-1}} & \dfrac{i}{C_{p-1}} \\ \dfrac{v_{C_1}}{L} & \dfrac{v_{C_2} - v_{C_1}}{L} & \cdots & \dfrac{v_{C_{p-1}} - v_{C_{p-2}}}{L} & \dfrac{E - v_{C_{p-1}}}{L} \end{bmatrix}$$

In order to use a continuous control theory, one should develop an average model in which all the signals are continuous. To obtain multicell converters average model, the instantaneous model control orders are replaced by their average values along one sampling period T_d.

$$\alpha_i = \frac{1}{T_d}\int_0^{T_d} u_i\, dt$$

This is only valid if time constants are much greater than the sampling period.

The general form of the *p*-cells converter average model can be written as [11].

$$\begin{cases} \dot{X}_1 = \dfrac{1}{C_1}(\alpha_1 - \alpha_2) X_p \\ \dot{X}_2 = \dfrac{1}{C_2}(\alpha_2 - \alpha_3) X_p \\ \vdots \\ \dot{X}_{p-1} = \dfrac{1}{C_{p-1}}\left(\alpha_{p-1} - \alpha_p\right) X_p \\ \dot{X}_p = -\dfrac{1}{L}(\alpha_2 - \alpha_1) X_1 - \dfrac{1}{L}(\alpha_3 - \alpha_2) X_2 \\ \qquad - \cdots - \dfrac{1}{L}\left(\alpha_p - \alpha_{p-1}\right) X_{p-1} - \dfrac{R}{L} X_p + \dfrac{E}{L}\alpha_p \end{cases} \tag{3}$$

where $\begin{cases} X = [V_{C_1}, V_{C_2}, \dots, V_{C_p}, I]^T \\ \alpha = \left[\alpha_1, \dots, \alpha_p\right]^T \end{cases}$.

III. Multicell Converter Hybrid Modes

III.1 Hybrid Systems Briefly

A hybrid system can be described by the interaction between a continuous dynamical system, whose behavior is described by continuous nonlinear differential equations, and by automata with discrete event dynamics behavior [12-13]. The hybrid model is completely described by the following system [9], [12].

$H = \{Q, X, \text{Init}, f, X_q, E, G\}$

where
- $Q = \{q_1, q_2, q_3, \ldots\}$ is the set of the discrete states (discrete locations);
- $X = R^n$ are the continuous states;
- $Init \subset Q{\times}X$ is a set of possible initial conditions;
- $f_{(.)}(.) : Q{\times}X \rightarrow R^n$ is the vector field associated with each discrete state;
- $X(.) : Q \rightarrow P(X)$ associates an invariant field for the discrete state q;
- $E \subset Q{\times}Q$ is the set of possible transitions in the automata;
- $G : E \rightarrow 2^X$ is the constraint in the continuous field for validating a transition $e \in E$;
- $R : G(e) \rightarrow P(X)$ is the continuous variables reinitialization relation at the time of a discrete transition.

III.2 Application to a Two-Cells Converter

This study is carried-out for a two-cells converter but could be easily generalized to a high number of cells [14]. The corresponding instantaneous model is then given by

$$\begin{cases} \dfrac{dv_C}{dt} = \dfrac{1}{C}(u_2 - u_1)\, i \\ \dfrac{di}{dt} = -\dfrac{R}{L} i - (u_2 - u_1)\dfrac{v_C}{L} + u_2 \dfrac{E}{L} \end{cases} \tag{4}$$

The considered converter is presented in Fig. 2. The continuous states vector is $x = [x_1\ x_2]^T$, where x_1 represents the floating voltage v_C and x_2 represent the load current i. Depending on the values of the discrete signals u_1 and u_2, four configurations are possible. Indeed, four operating modes can be distinguish and given by $Q = \{q_1, q_2, q_3, q_4\}$. Each mode is defined in the space of $X_{qi} = R^2$, $\forall q_i \in Q$. Here the continuous dynamics can be given for each mode in the following form

$$\dot{x} = f_q(x) = A(q)x + B(q)$$

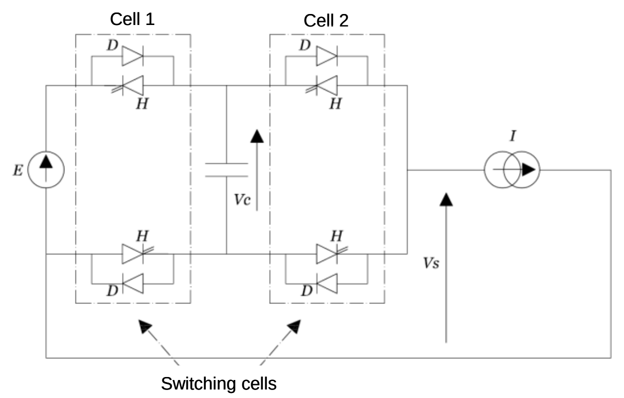


Fig. 2. Two-cells converter.

- Mode $q = q_1$ ($u_1 = 1$, $u_2 = 0$) (Fig. 3): The continuous variables dynamic equations are given by

$$f_{q_1}(x) = \begin{bmatrix} 0 & -\dfrac{1}{C} \\ \dfrac{1}{L} & -\dfrac{R}{L} \end{bmatrix} x \tag{5}$$

- Mode $q = q_2$ ($u_1 = 1$, $u_2 = 1$) (Fig. 4): The continuous variables dynamic equations are given by

$$f_{q_2}(x) = \begin{bmatrix} 0 & 0 \\ 0 & -\dfrac{R}{L} \end{bmatrix} x + \begin{bmatrix} 0 \\ \dfrac{E}{L} \end{bmatrix} \tag{6}$$

- Mode $q = q_3$ ($u_1 = 0$, $u_2 = 1$) (Fig. 5): The continuous variables dynamic equations are given by

$$f_{q_3}(x) = \begin{bmatrix} 0 & \dfrac{1}{C} \\ -\dfrac{1}{L} & -\dfrac{R}{L} \end{bmatrix} x + \begin{bmatrix} 0 \\ \dfrac{E}{L} \end{bmatrix} \tag{7}$$

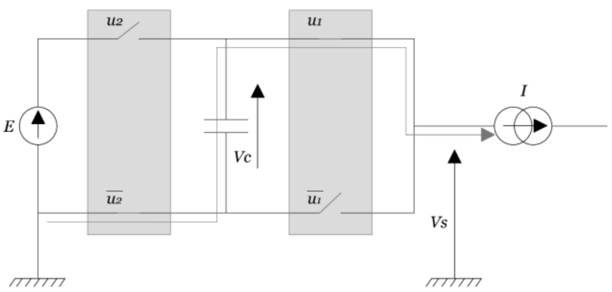


Fig. 3. Configuration *a*.

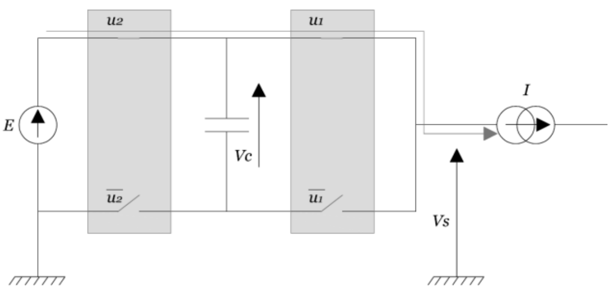


Fig. 4. Configuration *b*.

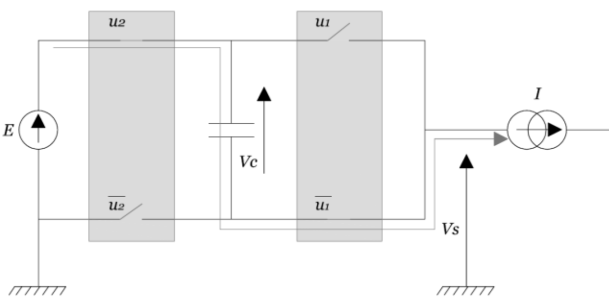


Fig. 5. Configuration *c*.

– Mode $q = q_4$ ($u_1 = 0$, $u_2 = 0$) (Fig. 6): The continuous variables dynamic equations are given by

$$f_{q_3}(x) = \begin{bmatrix} 0 & 0 \\ 0 & -\dfrac{R}{L} \end{bmatrix} x \qquad (8)$$

It is possible to switch (switching transition) from a mode to another one. In practice, there are however some constraints that reduce the number of admissible transitions. In order to minimize energy losses it is common to impose that at each possible switching transition, only a unique modification in the switches u_k is admitted [15]. With this constraint, two switching transitions are found. The switching transition conditions, from the q_i operating mode to the q_j operating mode are defined by (Fig. 7)

$$E = \{(q_i, q_j), \forall i \neq j, \text{ for } i, j = 1, \ldots, 4\} \qquad (9)$$

In order to control the converter states variables, the transition conditions T_{ij} must be developed to allow reference tracking. For that purpose, sliding modes are therefore used for control law synthesis.

IV. Hybrid Sliding Modes Control Law Synthesis

Sliding modes are obviously adopted as the two-cells converter has at least one discrete control variable.

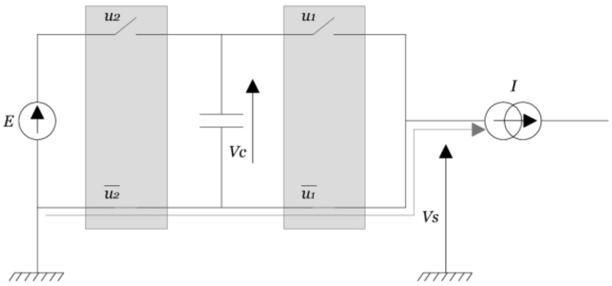


Fig. 6. Configuration *d*.

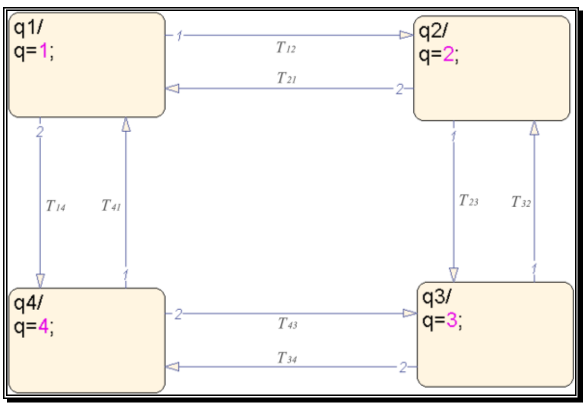


Fig. 7. Two-cells converter hybrid automata.

Indeed, the converter switches are binary controlled (0 or 1) [16-17].

Let v_{ref} and i_{ref} be the desired references of the output voltage and the load current, respectively. Let us define the tracking error

$$\Delta x^T = \begin{bmatrix} v_C - v_{ref} & i - i_{ref} \end{bmatrix} \qquad (10)$$

Where v_{ref} = E/2 satisfies the natural balancing.

Consider the following control sequences in closed-loop for the two-cells converter

$$u_i = \frac{1}{2}\left[1 - sign(S_i)\right] \quad i = 1, 2 \qquad (11)$$

where the sliding mode surfaces are given by

$$\begin{cases} S_1 = i_{ref} v_C - i v_{ref} \\ S_2 = i_{ref}(E - v_C) - i v_{ref} \end{cases} \qquad (12)$$

Then, the tracking error Δx is asymptotically stable.

Let us first show that the control objective is satisfied on the sliding surfaces *S*. If

$$\begin{cases} S_1 = 0 \\ S_2 = 0 \end{cases}$$

and using $v_{ref} = \dfrac{E}{2}$,

then we get, $\begin{cases} i_{ref} v_C - i v_{ref} = 0 \\ i_{ref}(E - v_C) - i v_{ref} = 0 \end{cases} \Rightarrow \begin{cases} v_C = v_{ref} \\ i = i_{ref} \end{cases}$

This is the proof that the surfaces S_1 and S_2 are attractive and invariant.

The proposed hybrid sliding mode control automata is illustrated by Fig. 12. The control aim is to make the switching surfaces converging to the origin, which therefore allow the state variables reaching their references.

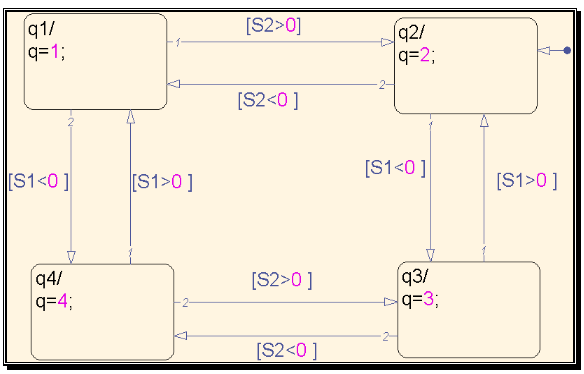


Fig. 8. Controller state flow.

V. Simulation Results

For the validation of the proposed hybrid sliding mode controller, simulations have been carried-out on a two-cells converter whose parameters are given in the Appendix ($x_0 = [0\ 0]^T$).

V.1 Simulation Results

Figures 9 and 10 show the floating voltage V_C and the load current I, respectively. With null initial conditions, V_C increases and stabilizes around its reference ($E/2$). It is obvious that the current dynamics is greater than the voltage one. This has led to strong ripples in the steady-state load current. This remark is also extended for the commutation surfaces S_1 and S_2 depicted in Fig. 11 and 12.

The main conclusion to be drawn is the proof of the proposed hybrid sliding mode control as are no steady-state errors in the states variables

Figure 13 illustrates the transitions evolution and shows that there are two stages. The first is the transient mode where the transition commutates between the q_3 and q_4 modes. The second stage is the permanent mode where the transitions jump-up between all modes.

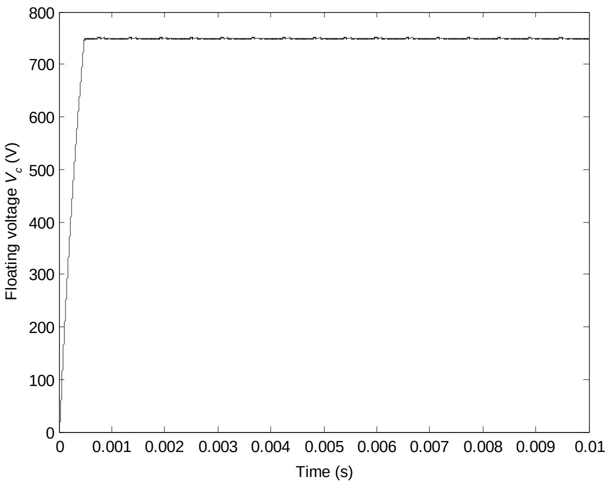


Fig. 9. Floating voltage.

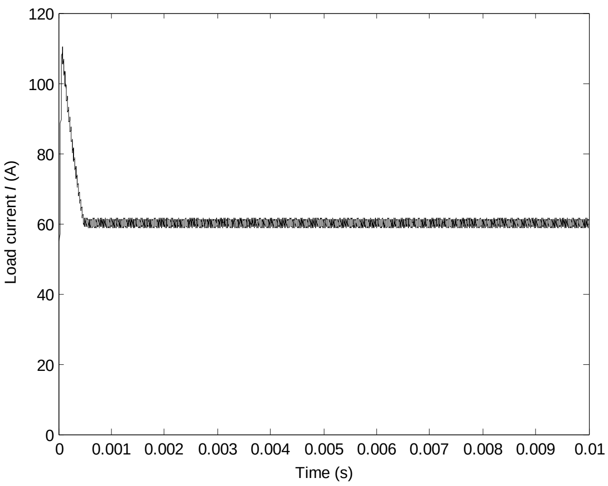


Fig. 10. Load current.

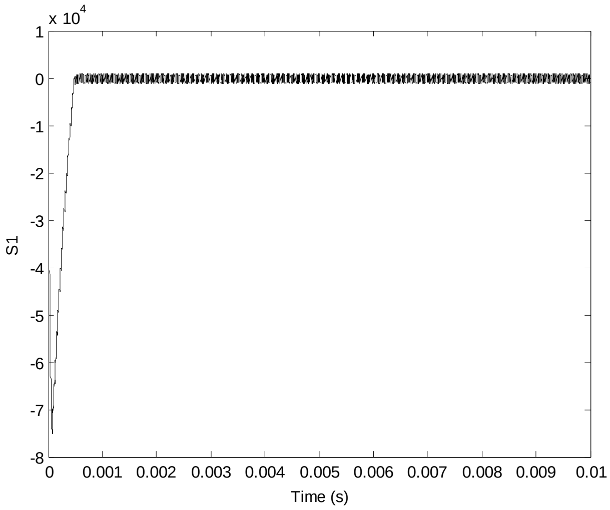


Fig. 11. Commutation function S_1.

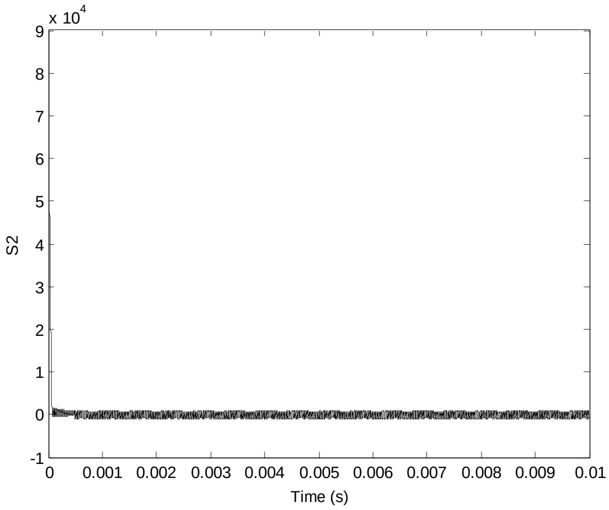


Fig. 12. Commutation function S_2.

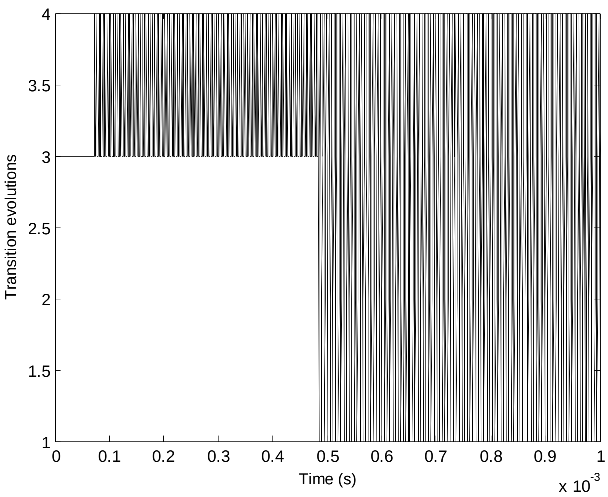


Fig. 13. Transitions.

Figure 10 shows that the transient current could reach a value close to twice its reference. This is a problem for a number of loads and should be carefully handled. Therefore, Fig. 14 illustrated the proposed modified automata.

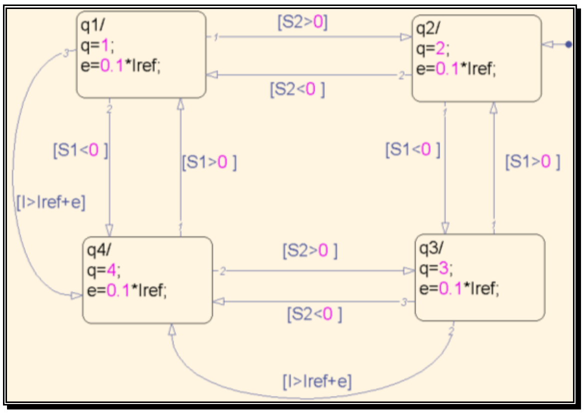


Fig. 14. Modified controller state flow.

In the above figure, it ca be seen that the q_4 mode is used to decrease the transient load current the reference one plus 10%. Figures 15 and 16 illustrate the use on the new automata.

V.2 Robustness Assessment

When modeling multicell converters, it is assumed that their parameters are well-known. Unfortunately, in real world, these parameters undergo changes over time mainly because of heating and aging. For this reason, it is necessary carry-out robustness tests against parameter variations.

V.2.1 Robustness versus power supply variation

Robustness test versus the power supply value changes are illustrated by Figs. 17 and 18. Indeed, at t = 0.005 s, E is slightly changed. According to these results, it is obvious that the control objective is fulfilled. Even, if the floating voltage has very slightly moved-up from its reference (zoom of Fig. 17).

V.2.2 Robustness versus load resistance variations

Robustness test versus the load resistance are illustrated by Figs. 19 and 20. Indeed, at t = 0.005 s, the load resistance is changed (less than 30%). These figures show that the state variables exhibit a transient but the promptly converge to their respective reference.

The above two tests confirm the robustness of the proposed hybrid sliding mode control strategy for the two-cells converter.

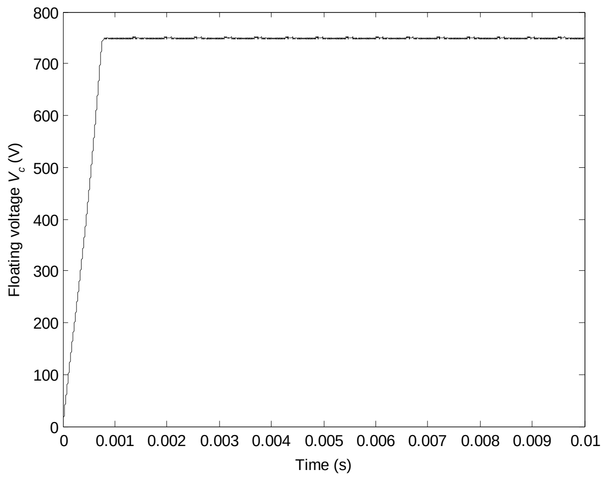


Fig. 15. Floating voltage.

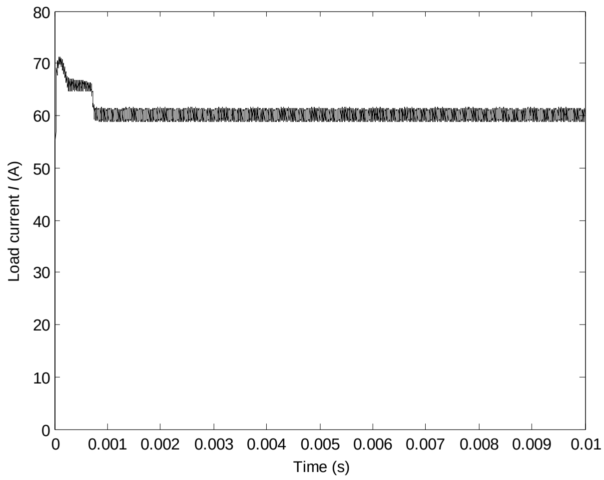


Fig. 16. Load current.

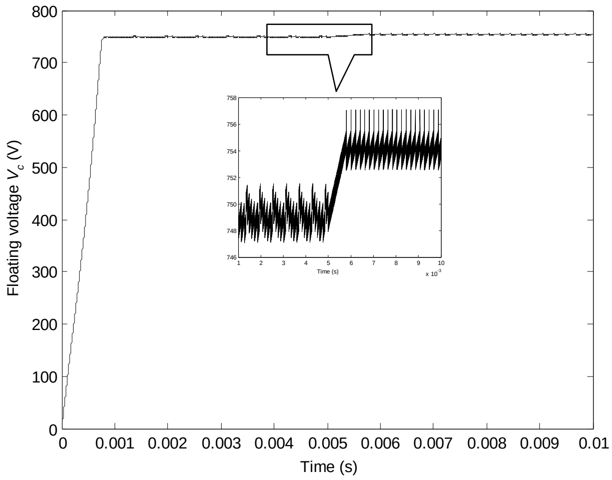


Fig. 17. Floating voltage.

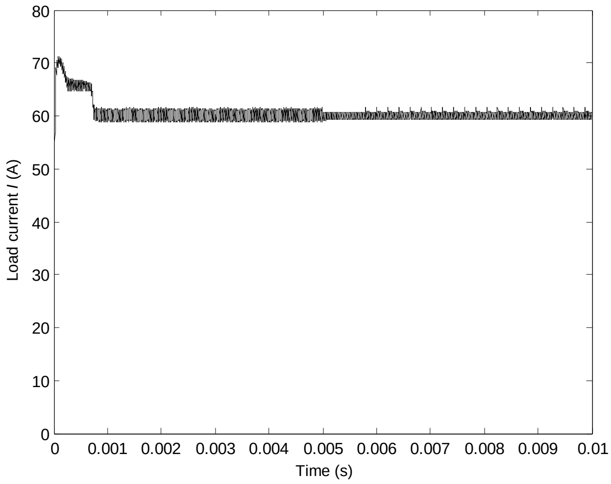


Fig. 18. Load current.

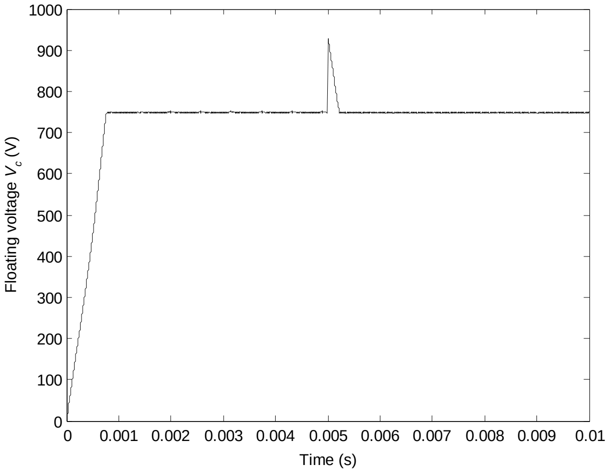


Fig. 19. Floating voltage.

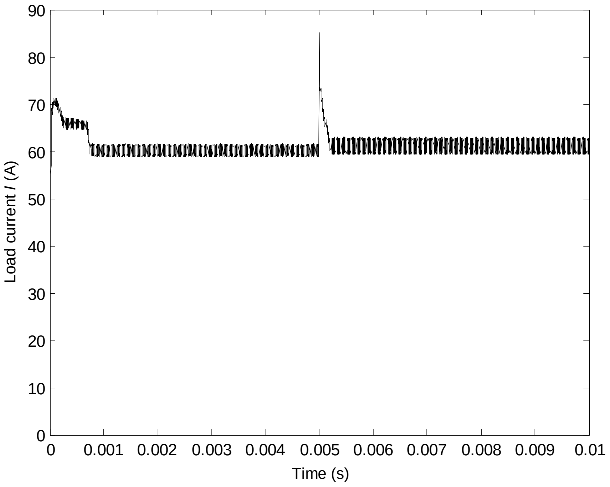


Fig. 20. Load current.

This is an extra justification of the sliding modes. Indeed, featuring robustness and high accuracy, they cope with system uncertainty keeping a properly chosen constraint by means of high-frequency control switching.

VI. Conclusion

This paper dealt with hybrid sliding mode control of multicell converters. It has been proposed hybrid modeling of a two-cells converter that was used to synthesize a hybrid sliding mode controller. The carried-out simulations show very promising results in terms of reference tracking performances and robustness. They prove the appropriateness of sliding mode control for such kind of hybrid system.

Appendix

PARAMETERS OF THE SIMULATED TWO-CELLS CONVERTER AND THE RL LOAD

C = 60 µF, E = 1500 V, v_{ref} = E/2, i_{ref} = 60 A R = 10 Ω, L = 0.5 mH

References

[1] T.A. Meynard, H. Foch, P. Thomas, J. Courault, R. Jakob and M. Nahrstaedt, "Multicell converters: basic concepts and industry applications," *IEEE Trans. Industrial Electronics*, vol. 49, n°5, pp. 955-964, October 2002.

[2] R. Stala, S. Pirog, M. Baszynski, A. Mondzik, A. Penczek, J. Czekonski and S. Gasiorek, "Results of investigation of multicell converters with balancing circuit—Part I," *IEEE Trans. Industrial Electronics*, vol. 56, n°7, pp. 2610-2619, July 2009.

[3] R. Stala, S. Pirog, A. Mondzik, M. Baszynski, A. Penczek, J. Czekonski and S. Gasiorek, "Results of investigation of multicell converters with balancing circuit—Part II," *IEEE Trans. Industrial Electronics*, vol. 56, n°7, pp. 2620-2628, July 2009.

[4] A. Ajami and M. Armaghan, "Vector control of induction motor drive based on mixed multi-cell cascaded inverter," *International Review on Modelling & Simulations*, vol. 3, n°5, pp. 767-774, October 2010.

[5] P. Ladoux, M. Machmoum, C. Batard, "Harmonic currents compensation for 1.5 kV DC railway substations," *International Review of Electrical Engineering*, vol. 4, n°3, pp. 380-391, June 2009.

[6] F. Ben Ammar and M. Ben Smida, "Modelling and control of a three-phase stacked multilevel voltage source inverter," *International Review of Electrical Engineering*, vol. 1, n°4, pp. 480-489, October 2006.

[7] A.K. Sadigh, S.H. Hosseini, M. Sabahi and G.B. Gharehpetian, "Double flying capacitor multicell converter based on modified phase-shifted pulsewidth mModulation," *IEEE Trans. Power Electronics*, vol. 259, n°6, pp. 1517-1526, June 2010.

[8] G. Gateau, M. Fadel, P. Maussion, R. Bensaid and T.A. Meynard, "Multicell converters: active control and observation of flying-capacitor voltages," *IEEE Trans. Industrial Electronics*, vol. 49, n°5, pp. 998-1008, October 2002.

[9] K. Benmansour, A. Benalia, M. Djemaï and J. de Leon, "Hybrid control of a multicellular converter," *Nonlinear Analysis: Hybrid Systems*, vol. 1, n°1, pp. 16-19, March 2007.

[10] R.H. Wilkinson, T.A. Meynard and H. du Toit Mouton, "Natural balance of multicell converters: The general case," *IEEE Trans. Power Electronics*, vol. 21, n°6, pp. 1658-1666, November 2006.

[11] D. Patino, P. Riedinger and C. Iung, "Predictive control approach for multicellular converters," *in Proceedings of the 2008 IEEE IECON*, pp. 3309-3314, November 2008.

[12] B. De Schutter, W.P.M.H. Heemels, J. Lunze and C. Prieur, *Survey of Modelling, Analysis and Control of Hybrid Systems*. in *HYCON Handbook of Hybrid Systems Control Theory-Tools-Applications*, Chapter 2, p. 31-55, Ed. F. Lamnabhi-Lagarrigue and J. Lunze, Cambridge University Press, 2009.

[13] J. Lygeros, K.H. Johansson, S.N. Simic, J. Zhang and S.S. Sastry, "Dynamical properties of hybrid automata," *IEEE Trans. Automatic Control*, vol. 48, n°1, pp. 2-17, January 2003.

[14] R.H. Wilkinson, T.A. Meynard and H. du Toit Mouton, "Natural balance of multicell converters: The two-cell case," *IEEE Trans. Power Electronics*, vol. 21, n°6, pp. 1649-1657, November 2006.

[15] B.P. McGrath, D.G. Holmes, "Optimal modulation of flying capacitor and stacked multicell converters using a state machine decoder," *IEEE Trans. Power Electronics*, vol. 22, n°2, pp. 508-516, March 2007.

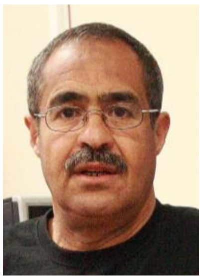
[16] M. Ghanes,F. Bejarano and J.P. Barbot, "On sliding mode and adaptive observers design for multicell converter," *in Proceedings of the 2009 IEEE ACC*, pp. 2134-2139, June 2009.

[17] A.M. Lienhardt, G. Gateau and T. Meynard, "Digital sliding-mode observer implementation using FPGA," *IEEE Trans. Industrial Electronics*, vol. 54, n°4, pp. 1865-1875, August 2007.

[1]University of Brest, EA 4325 LBMS, Rue de Kergoat, CS 93837, 29238 Brest Cedex 03, France (e-mail: Omar.Benzineb@univ-brest.fr, Mohamed.Benbouzid@univ-brest.fr).

[2]University of Blida, BP 192 Route de Soumaa, 09000 Bliba, Algeria.

[3]Electrical Engineering Department, Ecole National Supérieure Polytechnique, 10, Avenue Hassen Badi, BP 182, 16200 Algiers, Algeria.

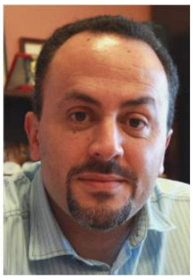


Omar Benzineb was born in Ksar El Boukhari, Algeria, in 1955. He received the B.Sc. degree from the Ecole Nationale Supérieure, Algiers, Algeria, in 1987, and the M.Sc. degree in Electrical Engineering from the University of Blida, Blida, Algeria, in 2004. In 1995, he joined the University of Blida, Blida, Algeria as a Teaching Assistant.

He is currently pursuing Ph.D. studies on fault detection and fault-tolerant control of electric machine drives.

Fateh Taibi was born in Algiers, Algeria, in 1985. He received the B.Sc. and M.Sc. degrees in electrical and computer engineering from the Ecole Nationale Polytechnique, Algiers, Algeria, in 2008 and 2011, respectively.

He is currently pursuing Ph.D. studies on multicell converters in wind energy application.



Mohamed El Hachemi Benbouzid was born in Batna, Algeria, in 1968. He received the B.Sc. degree in electrical engineering from the University of Batna, Batna, Algeria, in 1990, the M.Sc. and Ph.D. degrees in electrical and computer engineering from the National Polytechnic Institute of Grenoble, Grenoble, France, in 1991 and 1994, respectively, and the Habilitation à Diriger des Recherches degree from the University of Picardie "Jules Verne," Amiens, France, in 2000.

After receiving the Ph.D. degree, he joined the Professional Institute of Amiens, University of Picardie "Jules Verne," where he was an Associate Professor of electrical and computer engineering. Since September 2004, he has been with the Institut Universitaire de Technologie of Brest, University of Brest, Brest, France, where he is a Professor of electrical engineering. His main research interests and experience include analysis, design, and control of electric machines, variable-speed drives for traction, propulsion, and renewable energy applications, and fault diagnosis of electric machines.

Prof. Benbouzid is a Senior Member of the IEEE Power Engineering, Industrial Electronics, Industry Applications, Power Electronics, and Vehicular Technology Societies. He is an Associate Editor of the IEEE TRANSACTIONS ON ENERGY CONVERSION, the IEEE TRANSACTIONS ON INDUSTRIAL ELECTRONICS, the IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY, and the IEEE/ASME TRANSACTIONS ON MECHATRONICS.

Mohamed Salah Boucherit was born in Algiers, Algeria, in 1954. He received the B.Sc., the M.Sc. and Ph.D. degrees all in electrical engineering from the Ecole Nationale Supérieure, Algiers, Algeria, in 1980, 1988 and 1995, respectively.

Upon graduation, he joined the Electrical Engineering Department of Ecole Nationale Polytechnique where he is Professor of electrical engineering. His research interests are in the area of electrical drives and process control.

Mohamed Tadjine was born in Algiers, Algeria, in 1966. He received the Engineering degree from the Ecole Nationale Polytechnique, Algiers, Algeria, in 1990, the M.Sc. and Ph.D. degrees in automatic control from the National Polytechnic Institute of Grenoble, Grenoble, France, in 1991 and 1994, respectively. From 1995 to 1997, he was a Researcher at the University of Picardie, Amiens, France.

Since 1997, he is with the Department of Automatic Control of the Ecole Nationale Polytechnique, Algiers, Algeria, where he is currently a Professor. His research interests are in robust and nonlinear control.